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COUPLED OHARA-RUDY NUMERICAL MODEL FOR HEART ELECTRO-MECHANICS

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Abstract:
It is of enormous importance to have accurate electro-mechanical models for better understanding of various heart diseases such as heart failure, cardiac arrhythmia, and cardiomyopathy. Muscles in the body are activated by electrical signals, transmitted from the nervous system to muscle cells, affecting the cell membranes potentials. Current flow through cell membrane contains ion currents of molecules such as sodium (Na^+), potassium (K^+) and calcium (Ca^{2+}), which are critically important for muscle function. Calcium current and concentration inside the cell are the main cause of generating active stress within muscle fibers.

In order to make computational models feasible for applications of such complex system, we introduced a Kojic Transport Model (KTM) for modelling physical fields, by formulating a KTM finite element for electrophysiology. In order to calculate calcium current and concentration within the cell, we coupled the KTM and the OHara-Rudy (ORd) membrane model. Electrophysiology module is further coupled with muscle mechanics by widely used relation for heart muscle that connects calcium concentration and active stress along muscle fibers. This methodology has been built in our finite element software PAK.

Key words: Kojic Transport Model, OHara-Rudy model, electrophysiology, heart muscle mechanics, electro-mechanical simulation

1. Introduction

In modeling of coupled physical phenomena, a large number of unknowns is present and needs to be solved, but this problem is resolved applying the KTM methodology so we are solving smaller number of unknowns without losing all necessary physical properties and accuracy.

2. Formulation of the KTM finite element coupled with heart muscle mechanics

Kojic Transport Model finite element (Fig. 1) is developed to include different physical fields [1]. Muscles in the body are activated by electrical signals transmitted to muscle cells. The signals produce change of the cell membranes potentials, causing current through membrane I_{mem} [2],

$$I_{mem} = G_m \left(V_e^{in} - V_e^{out} \right) + C_m \left(\frac{\partial V_e^{in}}{\partial t} - \frac{\partial V_e^{out}}{\partial t} \right) + I_{ion} \quad (1)$$

where G_m and C_m are the membrane conductivity and capacitance, and V_e^{in}, V_e^{out} are electrical potentials in and outside cell. The ion currents I_{ion} involve the flow through membrane of all vital molecules, which depend on the membrane potential. Calcium concentrations within muscle cells, fundamental for the muscle contraction, are induced by changes of the membrane potential and it is directly related to the generation of active stress in muscle fibers [3],

$$\sigma_{act} = \frac{\left[Ca^{2+}\right]^n}{\left[Ca^{2+}\right]^n + C_{50}^n}\sigma_{\max}\left[1+\eta\left(\lambda-1\right)\right] \tag{2}$$

where σ_{act} is the active stress along the fiber, Ca^{2+} is calcium concentration; and C_{50}^n, $\sigma_{\max}$, η and λ are material parameters determined experimentally.

The balance equation of a finite element (velocity formulation) can be written in the form

$$\left(\frac{1}{\Delta t}\mathbf{M} + \Delta t\mathbf{K}\right)\Delta\mathbf{V}^{(i)} = \mathbf{F}^{ext} - \mathbf{F}^{\text{int}(i-1)} - \frac{1}{\Delta t}\mathbf{M}\left(\mathbf{V}^{(i-1)} - \mathbf{V}^{t}\right) \tag{3}$$

Muscle deformation includes active stress within the internal force vector, beside material stress.

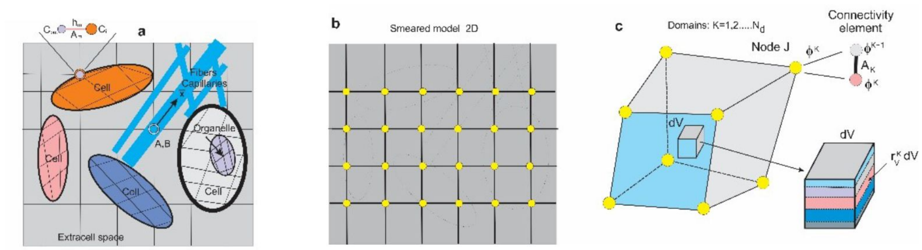

Fig. 1. KTM (or Composite Smeared Finite Element) finite element for gradient-driven field problems [1]. a) A detailed model with extracellular space, cells and organelles; b) FE mesh with KTM; c) The KTM Finite element with different domains and nodal 'degree of freedom, Φ^K, and connectivity element at node J between two domains.

3. Conclusions

A general KTM methodology for field problems is extended to include the electrical potential field and membrane ionic transport, that is very important in muscle and heart electro mechanics and also offer a computational tool for practical applications in biomedical investigations.

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